Impact of tracking system knowledge on multi-sensor 3D triangulation

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ABSTRACT

The three-dimensional (3D) location of objects can be determined using triangulation techniques with two or more passive, angles-only optical sensors. The present work describes how to determine necessary experimental parameters, such as camera resolution, pointing and location knowledge, and target pixel position estimation accuracy, in order to achieve a required 3D position accuracy. Two general types of accuracy requirements are examined, one in which absolute position of the object relative to an external coordinate system is required, and the second in which relative position of one object relative to a second object is required. While the basic approach can be applied to a wide variety of geometries and sensor-to-target ranges, the emphasis of this work is on outdoor applications involving long sensor-to-sensor baselines.

Keywords: triangulation, 3D position estimation, angles-only, multi-sensor, passive ranging, error propagation

1. INTRODUCTION

A single angles-only sensor (e.g., a standard imaging camera) does not provide range information. However, the use of two angles-only sensors allows one to use simple triangulation to determine range and hence the 3D location of an object [1-4]. Recent work has developed a general error analysis theory for two-sensor triangulation [5]. The general error analysis theory in [5] has been simplified and applied to the specific case of fixed sensors (both location and line-of-sight (LOS) pointing angles) for laboratory applications [6]. The current paper applies the general theory to examine triangulation problems involving tracking mounts in outdoor applications, typically involving long sensor-to-sensor baseline distances.

As described below, computation of the 3D location of a target requires knowledge of the (x,y,z) location of each sensor, as well as the LOS azimuth and elevation pointing angles (θ,ϕ) from each sensor to the target. Errors in knowledge of these ten parameters will result in errors in the 3D position estimate of the target. Furthermore, the relationship between measurement error and errors in the estimated target location are a function of the sensor-target-sensor geometry, with minimum error sensitivity occurring when the sensors are 90° apart and solutions becoming impossible when the sensors are 0° or 180° apart (as seen from the target).

The LOS to a target is usually not measured directly, but instead one measures the LOS of some specified boresight pixel on the focal plane and then measures the target location in pixels. Thus, in order to determine the LOS to the target, one must also know the Incremental Field Of View (IFOV) of an individual pixel. Therefore, we see that errors both in knowledge of the camera boresight pointing and errors in the target position estimation contribute to errors in the overall LOS knowledge and hence to the 3D position estimate.

The current work does not directly address a number of important contributors to the overall measurement error. Specifically, geometric lens distortion errors are considered to be part of the overall target location error in pixel coordinates. Atmospheric distortion will also introduce errors in the observed target location in pixel coordinates. For extended targets, one must determine corresponding target points in each image. An error in this selection (the correspondence problem) will also manifest itself as a LOS measurement error. This work also assumes a Euclidean coordinate system and errors resulting from curvature of the earth are not considered. The current work also does not address measurement techniques for determining sensor location and orientation or sources of error in those measurements.

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We begin by defining a coordinate system, writing the basic triangulation equations, and summarizing the error propagation equations. Next we examine how to determine the accuracy which can be obtained from a given instrument configuration. Finally, we examine how to design a test to achieve a required accuracy by flowing down an error budget. For both the given instrument configuration and the test design scenario, we examine both absolute position accuracy and relative position accuracy.

2. TWO SENSOR ERROR ANALYSIS

2.1 Triangulation

In order to perform triangulation using two angles-only sensors, we define a right-handed coordinate system with the x- and y-axes forming the horizontal plane and the z-axes pointing in the vertical direction. The azimuth angle ($\theta$) is measured clockwise toward the positive x-axis from the positive y-axis. Thus, the positive y-axis points to North and the positive x-axis points East. The elevation angle ($\phi$) increases from 0 in the x-y plane to a maximum of 90° when pointing vertically, parallel to the positive z-axis. We further define the horizontal range to the target, $r_i$, as the distance between the x-y components of the target location and the x-y components of the location of the $i^{th}$ sensor. The slant range from sensor $i$ to the target is $L_i$. The sensor separation angle is the angle measured from sensor one through the target to sensor two. These quantities are shown in Figure 1.

![Coordinate system definition](image)

The basic triangulation equations (from [5]) are:

\[
x_i = [x_2 \tan \theta_1 - x_1 \tan \theta_2 + (y_1 - y_2)\tan \theta_1 \tan \theta_2 \tan \theta_1 - \tan \theta_2]^{-1}
\]

(1)

\[
y_i = [y_1 \tan \theta_1 - y_2 \tan \theta_2 + x_2 - x_1 \tan \theta_1 - \tan \theta_2]^{-1}
\]

(2)

\[
r_i = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}
\]

(3)

\[
z_i = r_i \tan \varphi_i + z_i
\]

(4)

These equations relate the target location to measurements of the sensor position and LOS from the sensors to the target, allowing us to compute the target location from these measurements. The key question addressed in this paper is
determining the accuracy of that target position estimate. The subscripts ‘1’ and ‘2’ refer to sensors one and two, respectively.

The basic error propagation equations are derived in [5], assuming independent gaussian error statistics for each sensor and further assuming that the measurement statistics are identical in each axis (i.e., there is no preferred measurement direction). The results of reference [5] are complex sums of various partial derivatives that are simplified in reference [6] to the following equations:

$$\sigma_{x_1} = \sqrt{c_{x,pos}^2 \sigma_{pos}^2 + (rc_{x,\theta})^2 \sigma_\theta^2}$$  \hspace{1cm} (5)

$$\sigma_{y_1} = \sqrt{c_{y,pos}^2 \sigma_{pos}^2 + (rc_{y,\theta})^2 \sigma_\theta^2}$$  \hspace{1cm} (6)

$$\sigma_r = \sqrt{c_{r,pos}^2 \sigma_{pos}^2 + (rc_{r,\theta})^2 \sigma_\theta^2}$$  \hspace{1cm} (7)

$$\sigma_z = \sqrt{c_{z,r}^2 \sigma_r^2 + (rc_{z,\phi})^2 \sigma_\phi^2 + \sigma_{pos}^2}$$  \hspace{1cm} (8)

where the various $c$ parameters are error coefficients which are functions of the sensor-to-sensor separation angle. The standard deviation of the measurement errors are $\sigma_{pos}$ for the sensor position error, $\sigma_\theta$ for the azimuth error, and $\sigma_\phi$ for the elevation error. These equations relate measurement errors ($\sigma_{pos}$, $\sigma_\theta$, $\sigma_\phi$) to uncertainty in the target location ($\sigma_{xt}$, $\sigma_{yt}$, $\sigma_r$, $\sigma_z$). The $c$ values are computed in reference [6] from the equations in reference [5], and are presented graphically in reference [6]. The graphical results from [6] are reproduced in Figures 2 through 4.

Figure 2. Error propagation coefficients for equations (5), (6), relating sensor location errors in meters and LOS errors in degrees to target $(x,y)$ location estimate errors in meters.
Figure 3. Error propagation coefficient for equation (7) relating sensor location errors in meters and LOS errors in degrees to sensor to target horizontal range estimate errors in meters.

Figure 4. Error propagation coefficient for equation (8), relating sensor location errors in meters and LOS errors in degrees to target z position estimate errors in meters.

2.2 Error Tree

Estimates of the target location depend on measurements of the sensor location, the sensor LOS to the target, the geometry of the sensor locations relative to the target and each other, and timing accuracy of the measurements. The three sensor location measurements (x, y, z), and LOS measurements (θ, ϕ) for each sensor depend in turn on several additional measurements as shown in the error tree in Figure 5. The sum of all of the contributing errors results in the final measurement error for the sensor location and LOS.
The sensor location estimate depends on the accuracy of a position estimate (e.g., the location of a survey point) and the accuracy of the relationship between that point and the actual sensor focal plane location. The estimate of the LOS from the sensor to the target depends on accuracy of the knowledge of the sensor boresight LOS and of the LOS of the target relative to that boresight. These in turn depend on a number of other parameters as shown in Figure 5. The sensor-to-target geometry affects target location estimates as shown in Figures 2 through 4. Since we are primarily interested in position estimates of moving targets, and since our sensors may be changing position and LOS over time, timing accuracy of the measurements becomes important as well.

2.3 Absolute versus relative errors

Absolute position estimates are necessary when cueing one system with another. Consider the case in which a target passes a number of optical sensors in succession. The first two sensors observe the target and estimate its 3D location. This information is then used to cue the third sensor. In this case, the 3D target position accuracy should be within the field of view of the third sensor. Absolute position estimates may also be necessary when comparing the results of two different sensors, such as optical and radar. Alternatively, a 3D triangulation system used in targeting must be able to provide absolute coordinates relative to some other system, such as GPS. Absolute position estimates are affected by all measurement errors shown in the error tree (Figure 5).

Relative position estimates are used to estimate the trajectory of a single target or to estimate the position of one target relative to another. In the former case, we are often interested in estimating the velocity or acceleration of a target. Any measurement errors that cancel out (i.e., a constant offset in the estimated target position) will not affect the velocity or acceleration estimate and so are not important. Measurement errors that might cancel out are normally static measurement values such as sensor location for fixed sensors.
3. EVALUATION OF TRIANGULATION ACCURACY

In this example, we assume a typical test scenario and need to determine the expected accuracy of our results. This situation may arise when the sensor locations and parameters are dictated by other measurement requirements and triangulation for 3D position estimates is only a secondary consideration. In other cases, there may be physical constraints limiting the placement of sensors.

We consider a setup similar to that of Figure 1, in which the sensors are located at \( s_1 = (1000, 0, 500) \), \( s_2 = (3500, 500, 0) \), and target = \( (2000, 2000, 4000) \) with distances in meters. If we assume a 2° field of view (FOV) and a 1000x1000 pixel sensor, then the FOV of a pixel (the IFOV) is 34.9 \( \mu \text{rad} \). Given these basic parameters, we can compute the following values for LOS angles and ranges (table 1). The sensor-to-sensor separation angle is 71.6°.

<table>
<thead>
<tr>
<th>Sensor 1</th>
<th>Sensor 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location (x,y,z)</td>
<td>(1000, 0, 500)</td>
</tr>
<tr>
<td>Horizontal range, r (m)</td>
<td>2236.07</td>
</tr>
<tr>
<td>Slant range, L (m)</td>
<td>4153.31</td>
</tr>
<tr>
<td>Azimuth (degrees)</td>
<td>26.565</td>
</tr>
<tr>
<td>Elevation (degrees)</td>
<td>57.426</td>
</tr>
<tr>
<td>FOV at target (m)</td>
<td>145.04</td>
</tr>
<tr>
<td>IFOV at target (m)</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Table 1. Sensor Parameters

For a real system, one would determine the achievable measurement accuracy for position and LOS pointing from the system itself. For this example, we assume that the sensor position can be measured to within \( \sigma_{\text{pos}} = 0.5 \text{ m} \) and the boresight LOS to within 1 pixel (35 \( \mu \text{rad} \)). Since we are assuming long focal length lenses, we assume that geometric distortion is negligible or can be calibrated out during image processing. We assume a target detection and track accuracy of 0.5 pixels. The total LOS error is the quadrature sum of the boresight LOS error and the target-to-boresight LOS error, or \( \sigma_\theta = \sigma_\phi = 1.1 \text{ pixels} = 38.4 \mu \text{rad (0.0022°)} \). Using the sensor-to-sensor separation angle of 71.6° and referring to Figures 2 and 3, we look up the x, y, and horizontal range (r) error coefficients. Using an average elevation angle of 60° and referring to Figure 4, we look up the z error coefficients. These values are tabulated in Table 2 below.

<table>
<thead>
<tr>
<th>Coef.</th>
<th>c(x,pos)</th>
<th>c(y,pos)</th>
<th>c(x,LOS)</th>
<th>c(y,LOS)</th>
<th>c(r,pos)</th>
<th>c(r,LOS)</th>
<th>c(z, LOS)</th>
<th>c(z,r)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>1.047</td>
<td>1.047</td>
<td>0.018</td>
<td>0.018</td>
<td>1.515</td>
<td>0.02</td>
<td>0.07</td>
<td>1.732</td>
</tr>
</tbody>
</table>

Table 2. Error coefficients for 72° azimuth separation and 60° elevation angle.

Inserting the error coefficient values from Table 2 into equations (5)-(8), along with our measurement error estimates and the average horizontal range of 2178 m, we obtain the estimates for the triangulation error shown in Table 3. The estimate for the z position error is based on using a single sensor to compute z in equation (4). Using two sensors and averaging, we would expect the result to be \( \sqrt{2} \) smaller, or 1.027 m in the z dimension.

<table>
<thead>
<tr>
<th>Target parameter</th>
<th>Error (m) from position error, ( \sigma_{\text{pos}} )</th>
<th>Error (m) from LOS error, ( \sigma_{\text{LOS}} )</th>
<th>Total error (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_t )</td>
<td>0.524</td>
<td>0.086</td>
<td>0.531</td>
</tr>
<tr>
<td>( y_t )</td>
<td>0.524</td>
<td>0.086</td>
<td>0.531</td>
</tr>
<tr>
<td>( r )</td>
<td>0.758</td>
<td>0.096</td>
<td>0.764</td>
</tr>
<tr>
<td>( z_t )</td>
<td>1.414</td>
<td>0.335</td>
<td>1.453</td>
</tr>
</tbody>
</table>

Table 3. Target position error estimates.
For the given setup, the target position error is totally dominated by the sensor location error with negligible contribution from the LOS pointing error. If our sensors are mounted on mobile platforms (such as aircraft or vehicles), then the actual error resulting from both position and LOS measurements changes with each measurement. In addition, the total error estimate must be applied both to our absolute position estimates and to relative measurements of frame-to-frame position for a single target, as well as single frame position of one target relative to another. On the other hand, if the sensors are at fixed locations (such as fixed tracking mounts), the errors in the sensor location provide a constant bias in the absolute position but do not contribute to either frame-to-frame errors for a single target or to single-frame separation estimates for two targets. In this case, the relevant target position error drops from the total error of approximately 0.5 m, to a mere 90 cm in the x and y dimension and 0.3 m in the z dimension. One might ask how the LOS error in the x and y dimension (86 mm) can be less than the projected pixel size (150 mm). This is because the projected pixel is a distance L (the total slant range) away from the sensor while the x and y errors use only the horizontal range, r.

If we express the position estimation accuracy using a normalized error of the target position error divided by the slant range distance, we obtain a normalized error of 0.00012 or 0.012%. Of course this is based on the assumed measurement accuracies which may or may not be achievable for a real system. Also, with a separation angle of 72°, this setup has close to an ideal, minimal error observation geometry. Nevertheless, this demonstrates the technique and considerations when evaluating the accuracy of a specific sensor setup.

4. FROM ACCURACY REQUIREMENTS TO TEST SETUP

In this section, we begin with a basic measurement requirement and flow down an error budget to design an observation system. The task is to measure the relative position of several objects at an altitude of 100 km, to an accuracy of 1 m, using 2 fixed-location, ground-based sensors. We are further constrained by the accuracy of an external cueing system to a sensor FOV of 0.25° to ensure that the targets will actually be in the FOV of our sensors. We have an absolute target position accuracy requirement of 4 m to allow cueing of other precision sensors. We are constrained by technology to 1000x1000 pixel sensors. We accept a 0.5 pixel tracking accuracy as given. In order to simplify communications, we wish to select the shortest sensor-to-sensor baseline consistent with the rest of the requirements. We assume the same basic measurement accuracy as in the previous section, sensor location errors of 0.5 m, and boresight LOS errors of 1 pixel, resulting in a total LOS error of 1.1 pixels or 4.800 µrads.

The driving requirement in this specification will be the 1 m relative position accuracy in the vertical (z) direction. Turning to equation (8), we see that the relative position accuracy will be given by:

$$\sigma_{z_i} = r c_{z,\phi,\rho} \sigma_{\phi,\rho} / \sqrt{2}$$  \hspace{1cm} (9)

where the $\sqrt{2}$ results from averaging the z position estimate from 2 sensors. The other two terms in equation (8) depend only on static sensor position measurement errors and hence contribute only a constant bias offset in the position estimate. From simple geometry, we can write $r = h / \tan \phi$, where h is the target altitude (100 km). Since we want our estimated position accuracy to be no more than 1 m, we replace the equals sign in equation (8) with a less than sign. Substituting for r, and solving for c, we get:

$$\frac{\sigma_{z_i}}{\sigma_{\phi,\rho}} h \sqrt{2} \tan \phi < c_{z,\phi,\rho}$$  \hspace{1cm} (10)

Figure 4 shows a plot of $c_{z,\phi}$ as a function of elevation angle. To find the greatest elevation angle (and hence minimum horizontal range r), which satisfies equation (10), we plot the values from the left hand side of equation (10) on the same graph as the computed values of $c_{z,\phi}$ and find the intersection point of the two curves. This is shown in Figure 6.
The intersection point is at 70° which corresponds to a horizontal range \( r = 36,397 \) m.

Given this horizontal range value for the sensors, we can now turn to equations (5) and (6) to determine the minimum azimuth separation angle. As with the vertical error, we can drop the sensor position error term in equations (5) and (6) as a static bias error. In this case, the two equations reduce to:

\[
\sigma_{x_i} = r c_{x,\theta} \sigma_\theta
\]

with an identical equation for the \( y \) error. Plugging in the known values for the required target position error, the horizontal sensor to target range, and the LOS measurement error, we compute a value for the error coefficient of \( c_{x,0} = 0.100 \text{ deg}^{-1} \), which can be achieved with a separation of only 10°.

Next we must determine if this separation angle will also meet the absolute position accuracy requirement. It turns out that, using these values, we easily meet all of the requirements except the vertical (\( z \)) direction absolute error requirement (due to the dependence in equation (8) on \( \sigma_r \), which has a strong dependence on the azimuth separation angle). In order to meet the \( z \) direction absolute position requirement we increase the azimuth separation angle to 20°.

We confirm that we now meet the absolute position accuracy requirement by tabulating the error coefficients and computing values for equations (5)-(8). The basic geometry we have selected is a sensor azimuth separation angle of 20°, a horizontal sensor range of 36,397 m, and an elevation angle of 70°. With these values, we can look up the error coefficients shown in Table 4. The results are given in Table 5.

<table>
<thead>
<tr>
<th>Coef. Value</th>
<th>( c(x, \text{pos}) )</th>
<th>( c(y, \text{pos}) )</th>
<th>( c(x, \text{LOS}) )</th>
<th>( c(y, \text{LOS}) )</th>
<th>( c(r, \text{pos}) )</th>
<th>( c(r, \text{LOS}) )</th>
<th>( c(z, \text{LOS}) )</th>
<th>( c(z, r) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.799</td>
<td>2.799</td>
<td>0.048</td>
<td>0.048</td>
<td>3.364</td>
<td>0.056</td>
<td>0.149</td>
<td>2.747</td>
<td></td>
</tr>
</tbody>
</table>

Table 4. Error coefficients for 20° azimuth separation and 70° elevation angle.

<table>
<thead>
<tr>
<th>Target parameter</th>
<th>Error (m) from position error, ( \sigma_{\text{pos}} )</th>
<th>Error (m) from LOS error, ( \sigma_{\text{LOS}} )</th>
<th>Total error (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_i )</td>
<td>1.400</td>
<td>0.480</td>
<td>1.480</td>
</tr>
<tr>
<td>( y_i )</td>
<td>1.400</td>
<td>0.480</td>
<td>1.480</td>
</tr>
<tr>
<td>( r )</td>
<td>1.682</td>
<td>0.561</td>
<td>1.773</td>
</tr>
<tr>
<td>( z )</td>
<td>4.896</td>
<td>1.491</td>
<td>5.118</td>
</tr>
</tbody>
</table>

Table 5. Target position error estimates.
The x and y relative error (due to LOS errors) is 0.480 m which easily meets the 1 m requirement, while the total error is 1.480 m, which easily meets the 4 m requirement. Recall that the z error estimates are for single sensors in equation (4), so if we average two sensors, we divide the numbers in table 5 by $\sqrt{2}$, giving a relative z error of 1.054 m and an absolute z error 3.619 m.

For a different example, reference [6] solves an error flow-down problem in which the elevation angle is essentially zero.

5. SUMMARY

The purpose of this paper was to demonstrate how to apply the theoretical error propagation results of reference [5] to practical triangulation problems. We began by reviewing the basic triangulation equations and error propagation. Simplified error propagation equations were presented along with graphical error coefficients. This was followed by a brief discussion of error sources contributing to the basic sensor position and LOS measurements.

We first applied these results to estimate the accuracy that could be obtained from a typical outdoor triangulation experiment. In this case, we used sensors approximately 4 km from the target with good viewing geometry and reasonable measurement accuracy assumptions to estimate a target position accuracy of approximately 0.5 m. Next we began with a target position estimation accuracy requirement and demonstrated a method for flowing down the error budget to design an observation system.

REFERENCES